

## REVIEW SESSION ON COMPLEX ANALYSIS

MATH 241

These problems are definitely above the average level, I want to make it a little hard to motivate you to review, and design a lot of traps to help you discover the possible mistakes. So do not be anxious too much about it. Work hard and take it easy!

1. Write down the Maclaurin expansion of  $\frac{1}{1+e^z}$  up to degree 2. What is the radius of convergence of its Maclaurin expansion?
2. Sketch the domain for  $z$  on which  $e^{2z}$  has a positive real part.
3. For the vector field  $V = (x + 2y, y)$ , what are the circulation and the net flux around the unit circle  $\{z \mid |z| = 1\}$ ?
4. Calculate  $P.V. \int_{-\infty}^{\infty} \frac{1}{x^4+x^2+1} dx$ .
5. Find all the solutions to  $z^3 + (z + 2)^3 = 0$ .
6. Let  $C$  be the ellipse  $4x^2 + y^2 = 9$ , oriented clockwise. Calculate  $\oint_C \frac{\sin z}{z^2+4} dz$ .
7. Write down the Laurent expansion of  $\frac{z+1}{(z-5)^2}$  from degree  $-4$  to  $+4$  on: (1)  $\{z \mid 0 < |z - 5| < 10\}$ ; (2)  $\{z \mid 6 < |z| < 8\}$ .
8. What is the set where  $f(x + iy) = (x^2 - 2y) + i(\frac{2}{3}x^3 + xy)$  is differentiable? analytic?